Int J Theor Phys (2008) 47: 1286–1293
DOI 10.1007/s10773-007-9562-4

New Exact Solutions of the (2+1)-Dimensional Asymmetric Nizhnik-Novikov-Veselov System

Chao-Qing Dai · Sheng-Sheng Wu · Xu Cen

Received: 26 July 2007 / Accepted: 11 September 2007 / Published online: 2 October 2007

Abstract Using symbolic and algebra computation, the extended tanh-function method (ETM) based on mapping method is further extended. The new variable separation solutions of the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov (ANNV) system are derived.

Keywords Improved mapping method · ANNV system · Exact solutions

1 Introduction

In last two decades, increasing attention has been paid to the study of the solution theory in many natural science such as chemistry, biology, mathematics, communication and particularly in almost all branches of physics like the fluid dynamics, plasma physics, field theory, nonlinear optics and condensed matter physics, etc. How to derive possible exact solutions to a nonlinear model arising from the field of mathematical physics is a popular topic, but solving nonlinear physics problems is much more difficult than solving the linear ones.

As one of the effective methods in linear physics, the variable separation approach (VSA) has been successfully extended to nonlinear domains. The MLVSA has also been established for various (2+1)-dimensional models [1]. Recently, along with linear variable separation idea and using the extended tanh-function method (ETM) based on mapping method, Zheng and some authors realized variable separation for some (2+1)-dimensional systems, such as Broer-Kaup-Kupershmidt (BKK) system [2], Kortweg-de Vries (KdV) equation [3] and asymmetric Nizhnik-Novikov-Veselov (NNV) system [4], etc. Moreover, some authors [5, 6] successfully generalized ETM to the (1+1)-dimensional and (3+1)-dimensional nonlinear physical models. However, in fact, various solutions including solitary wave solutions, periodic wave solutions and rational function solutions derived by ETM in [2–6], which seem independent, depend on each other. This viewpoint has been proven in [7]. Then, by the non-standard truncated expansion, Fang and his groups also generalized the ETM and

C.-Q. Dai (✉) · S.-S. Wu · X. Cen
School of Sciences, Zhejiang Forestry University, Linan 311300, People's Republic of China
e-mail: dcq424@126.com

obtained various so-called new symmetrical variable separation solutions for the (2+1)-dimensional BKK [8], Boiti-Leon-Pempinelli (BLP) [9] and NNV [10] systems, etc. However, similarly to our report in [7], we have also proven in [11] that these so-called new solutions in [8–10] also depend on each other and the effective solution is identical to the universal formula in [1], which has been given in [7].

More recently, a mapping method [12] is used to realize variable separation of the (2+1)-dimensional dispersive long wave equation (DLWE). We have also successfully generalized the projective Riccati equation method (PREM) to derive variable separation solutions for some (2+1)-dimensional systems [13, 14]. Moreover, with the help of q-deformed hyperbolic functions, we have also successfully obtained a new variable separation solution for the (2+1)-dimensional Kortweg-de Vries (KdV) equation [15]. Now a natural and important issue is whether we can derive new variable separation solutions via other direct methods. Motivated by this question, we further extend the mapping method and obtain new variable separation solutions of following (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov (ANNV) system

$$u_t + u_{xxx} - 3(uv)_x = 0, \qquad v_y = u_x. \tag{1}$$

Equation (1) was first derived by Boiti et al. [16] using the weak Lax pair. In [17] and may be considered as a model for an incompressible fluid where u and v are the components of the (dimensionless) velocity [18]. The spectral transformation for this system has been investigated in [16, 19]. This system has also been considered in [20] as a generalization to (2+1)-dimensions of the results from Hirota and Satsuma [21]. Moreover, Hu et al. used the Darboux transformations (DT) to find the variable separation solutions of this system in [22].

The paper is organized as follows. In Sect. 2, the extended tanh-function method (ETM) is reviewed, and the l-deformed functions are introduced. The variable separation solutions of (2+1)-dimensional ANNV system are obtained in Sect. 3. A brief discussion and summary is given in the last section.

2 Further Extended Tanh-Function Method

In the following we would like to outline the main steps of our method:

For a given nonlinear partial differential equation (NPDE) with independent variables $x = (x_0 = t, x_1, x_2, \ldots, x_m)$, and dependent variable u,

$$L(u, u_t, u_{x_i}, u_{x_i x_j}, \ldots) = 0, \tag{2}$$

where L is in general a polynomial function of its argument, and the subscripts denote the partial derivatives. One assumes that (2) possesses the following ansatz

$$u = a_0(x) + \sum_{j=1}^{n} \left\{ a_j(x)\phi[R(x)]^j + b_j(x)\phi[R(x)]^{j-1}\sqrt{l_1 + l_2\phi[R(x)]^2} \right\}, \tag{3}$$

with the Riccati equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}R} = l_1 + l_2\phi^2, \tag{4}$$

where $a_0(x)$, $a_j(x)$, $b_j(x)$ and $R(x)$ are arbitrary functions of $x = (x_0 = t, x_1, x_2, \ldots, x_m)$ to be determined, l_1 and l_2 are two real constants, and n is fixed by balancing the linear term of the highest order with the nonlinear term in (2). To determine u explicitly, one may substitute (3) and (4) into the given NPDE, collect the coefficients of the polynomials in ϕ and $\sqrt{l_1 + l_2\phi[R(x)]^2}$, then eliminate each coefficient to derive a set of partial differential equations of $a_0(x)$, $a_j(x)$, $b_j(x)$ and $R(x)$, solve this system of partial differential equations to obtain $a_0(x)$, $a_j(x)$, $b_j(x)$ and $R(x)$. Finally, as (4) possesses the general solution (without loss of generality, here we only consider the case of $l_1 > 0$).

(i) When $l_1 l_2 = -1$,

$$\phi_1 = l_1 \tanh_{l_1}(R), \tag{5}$$

$$\phi_2 = l_1 \coth_{l_1}(R). \tag{6}$$

(ii) When $l_1 l_2 = 1$,

$$\phi_3 = l_1 \tan_{l_1}(R), \tag{7}$$

$$\phi_4 = l_1 \cot_{l_1}(R). \tag{8}$$

(iii) When $l_1 = 0$,

$$\phi_5 = -\frac{1}{l_2 R}. \tag{9}$$

Moreover, (4) has combined solutions:

(iv) When $l_1 = -l_2 = \frac{1}{2}$,

$$\phi_6 = \tanh(R) \pm \mathrm{i}\,\mathrm{sech}(R), \tag{10}$$

$$\phi_7 = \coth(R) \pm \mathrm{csch}(R). \tag{11}$$

(v) When $l_1 = l_2 = \pm\frac{1}{2}$,

$$\phi_8 = \tan(R) \pm \sec(R), \tag{12}$$

$$\phi_9 = \cot(R) \pm \csc(R). \tag{13}$$

One substitutes $a_0(x)$, $a_j(x)$, $b_j(x)$, $R(x)$ and (5–13) into (3), and obtains exact solutions of the given NPDE in concern.

Functions in (5–8) are l-deformed functions [23], whose properties will be recalled, that is,

$$\begin{aligned} &\sinh_l(R) = \frac{e^R - l e^{-R}}{2}, \qquad \cosh_l(R) = \frac{e^R + l e^{-R}}{2}, \\ &\tanh_l(R) = \frac{\sinh_l(R)}{\cosh_l(R)}, \qquad \mathrm{sech}_l(R) = \frac{1}{\cosh_l(R)}, \quad R \in \mathbf{C}. \end{aligned} \tag{14}$$

It is straightforward to see that the following formulas hold,

$$\begin{aligned} &(\sinh_l(R))' = \cosh_l(R), \qquad (\cosh_l(R))' = \sinh_l(R), \\ &\cosh_l^2(R) - \sinh_l^2(R) = l, \qquad (\tanh_l(R))' = l\,\mathrm{sech}_l^2(R), \\ &(\mathrm{sech}_l(R))' = -\tanh_l(R)\mathrm{sech}_l(R), \qquad \tanh_l^2(R) = 1 - l\,\mathrm{sech}_l^2(R). \end{aligned} \tag{15}$$

Correspondingly, we can define l-deformed triangular functions as follows:

$$\sin_l(R) = \frac{e^{\mathrm{i}R} - le^{-\mathrm{i}R}}{2\mathrm{i}}, \qquad \cos_l(R) = \frac{e^{\mathrm{i}R} + le^{-\mathrm{i}R}}{2}, \\ \tan_l(R) = \frac{\sin_l(R)}{\cos_l(R)}, \qquad \sec_l(R) = \frac{1}{\cos_l(R)}. \tag{16}$$

They satisfy the following formulas:

$$(\sin_l(R))' = \cos_l(R), \qquad (\cos_l(R))' = -\sin_l(R), \\ (\tan_l(R))' = l\sec_l^2(R), \qquad (\sec_l(R))' = \tan_l(R)\sec_l(R), \\ \cos_l^2(R) + \sin_l^2(R) = l, \qquad 1 + \tan_l^2(R) = l\sec_l^2(R). \tag{17}$$

Remark 1 In [2–7], authors assume the ansatz of (2) has the form

$$u = a_0(x) + \sum_{j=1}^{n} a_j(x)\phi(R(x))^j,$$

which is merely special case of our ansatz (3) when $b_j(x) = 0$. Therefore, we can obtain more new exact solutions of NPDE by the ansatz (3) in present paper.

Remark 2 The similar ansatz (3) has been introduced by Li et al. in [24], however, they merely obtained the travelling solutions by this ansatz. Obviously, our ansatz (3) has more universal form due to the arbitrary function R of $x = (x_0 = t, x_1, x_2, \ldots, x_m)$. Moreover, the Ricatti equation in this paper has a more universal form than the one in [24].

3 Variable Separation Solutions for the (2+1)-Dimensional ANNV System

Now we apply the improved mapping method in Sect. 2 to the (2+1)-dimensional ANNV system (1). By the balancing procedure, ansatz (3) becomes

$$u = a_0 + a_1\phi(R) + b_1\sqrt{l_1 + l_2\phi(R)^2} + a_2\phi(R)^2 + b_2\phi(R)\sqrt{l_1 + l_2\phi(R)^2}, \tag{18}$$

$$v = c_0 + c_1\phi(R) + d_1\sqrt{l_1 + l_2\phi(R)^2} + c_2\phi(R)^2 + d_2\phi(R)\sqrt{l_1 + l_2\phi(R)^2}, \tag{19}$$

where $R \equiv R(x, y, t)$, $a_i \equiv a_i(x, y, t)$, $c_i \equiv c_i(x, y, t)$ $(i = 0, 1, 2)$, $b_j \equiv b_j(x, y, t)$, $d_j \equiv d_j(x, y, t)$ $(j = 1, 2)$, and ϕ satisfies (4). Inserting (18) and (19) with (4) into (1), selecting the variable separation ansatz

$$R = p(x, t) + q(y, t), \tag{20}$$

and eliminating all the coefficients of polynomials of ϕ and $\sqrt{l_1 + l_2\phi(R)^2}$, one gets a set of partial differential equations

$$-12a_2c_2p_xl_2 - 12b_2d_2p_x{l_2}^2 + 24a_2{p_x}^3{l_2}^3 = 0, \tag{21}$$

$$6a_1{l_2}^3{p_x}^3 - 9b_2p_xd_1{l_2}^2 + 18a_{2,x}{p_x}^2{l_2}^2 - 9b_1d_2p_x{l_2}^2 - 3a_2c_{2,x} - 3b_2d_{2,x}l_2 \\ - 9a_2c_1p_xl_2 - 9a_1c_2p_xl_2 - 3a_{2,x}c_2 - 3\,b_{2,x}d_2l_2 + 18a_2p_xp_{x,x}{l_2}^2 = 0, \tag{22}$$

$$-3b_{2,x}d_1l_2+6a_{1,x}l_2{}^2p_x{}^2-3a_2c_{1,x}+2a_2l_2q_t+6a_1l_2{}^2p_xp_{x,x}-3a_{1,x}c_2$$
$$-6a_2p_xl_2c_0-12a_2c_2p_xl_1-3a_1c_{2,x}-3b_{1,x}d_2l_2-6b_1d_1l_2{}^2p_x-3b_2d_{1,x}l_2$$
$$+2a_2l_2p_t-3a_{2,x}c_1-3b_1d_{2,x}l_2+2a_2p_{x,x,x}l_2-18b_2d_2p_xl_1l_2+40a_2p_x{}^3l_2{}^2l_1$$
$$-6a_1c_1p_xl_2+6a_{2,x}p_{x,x}l_2+6a_{2,x,x}p_xl_2-6a_0c_2p_xl_2=0, \tag{23}$$

$$a_1p_{x,x,x}l_2+a_1l_2q_t+3a_{1,x}p_{x,x}l_2-3b_{2,x}d_2l_1-3b_1d_{1,x}l_2-3b_{1,x}d_1l_2+a_1l_2p_t$$
$$+3a_{1,x,x}p_xl_2-3b_2d_{2,x}l_1-3a_1p_xl_2c_0-9a_1c_2p_xl_1-3a_0c_1p_xl_2-9a_2c_1p_xl_1$$
$$+24a_{2,x}p_x{}^2l_1l_2+8a_1l_2{}^2p_x{}^3l_1-3a_{2,x}c_0-3a_{1,x}c_1-3a_0c_{2,x}-3a_2c_{0,x}$$
$$-3a_{0,x}c_2-3a_1c_{1,x}+a_{2,t}+a_{2,x,x,x}+24a_2p_xp_{x,x}l_1l_2-12b_1d_2p_xl_1l_2$$
$$-12b_2p_xd_1l_1l_2=0, \tag{24}$$

$$-3a_{0,x}c_1-3a_0c_{1,x}+2a_2l_1q_t-6a_0c_2p_xl_1+16a_2p_x{}^3l_2l_1{}^2+2a_2l_1p_t-3a_1c_{0,x}$$
$$-6a_1c_1p_xl_1+a_{1,t}+a_{1,x,x,x}-6b_2d_2p_xl_1{}^2+6a_1l_2p_xp_{x,x}l_1-3b_{2,x}d_1l_1$$
$$+2a_2p_{x,x,x}l_1+6a_{2,x,x}p_xl_1-3a_{1,x}c_0-6a_2p_xl_1c_0-3b_1d_{2,x}l_1-3b_2d_{1,x}l_1$$
$$-6b_1d_1l_2p_xl_1+6a_{1,x}l_2p_x{}^2l_1+6a_{2,x}p_{x,x}l_1-3b_{1,x}d_2l_1=0, \tag{25}$$

$$3a_{1,x}p_{x,x}l_1+6a_2p_xp_{x,x}l_1{}^2-3a_0c_{0,x}+a_{0,t}+3a_{1,x,x}p_xl_1+a_1p_{x,x,x}l_1+a_1l_1q_t$$
$$-3a_0c_1p_xl_1+a_{0,x,x,x}-3a_{0,x}c_0+6a_{2,x}p_x{}^2l_1{}^2+2a_1l_2p_x{}^3l_1{}^2-3b_{1,x}d_1l_1$$
$$-3b_1d_{1,x}l_1-3b_1d_2p_xl_1{}^2-3b_2p_xd_1l_1{}^2+a_1l_1p_t-3a_1p_xl_1c_0=0, \tag{26}$$

$$-12b_2c_2p_xl_2-12a_2d_2l_2p_x+24b_2l_2{}^3p_x{}^3=0, \tag{27}$$

$$-9a_1d_2l_2p_x-3a_2d_{2,x}+18b_2l_2{}^2p_{x,x}p_x-9a_2d_1l_2p_x-3b_2c_{2,x}-9b_1c_2p_xl_2$$
$$-3b_{2,x}c_2-9b_2c_1p_xl_2+18b_{2,x}l_2{}^2p_x{}^2-3a_{2,x}d_2+6b_1l_2{}^3p_x{}^3=0, \tag{28}$$

$$-3b_2c_{1,x}-6b_2l_2p_xc_0-3b_1c_{2,x}+6b_{1,x}l_2{}^2p_x{}^2+2b_2l_2q_t-3a_{2,x}d_1+6b_{2,x,x}l_2p_x$$
$$-3a_1d_{2,x}-9a_2p_xl_1d_2-3b_{1,x}c_2+6b_1l_2{}^2p_{x,x}p_x-3a_{1,x}d_2+28b_2p_x{}^3l_2{}^2l_1$$
$$+2b_2l_2p_t-6a_0d_2l_2p_x-6a_1d_1l_2p_x-3b_{2,x}c_1-6b_1c_1p_xl_2+2b_2l_2p_{x,x,x}$$
$$+6b_{2,x}l_2p_{x,x}-3a_2d_{1,x}-9b_2c_2p_xl_1=0, \tag{29}$$

$$-3a_0d_{2,x}+15b_2p_{x,x}l_2p_xl_1-3a_{1,x}d_1-3b_2c_{0,x}-3a_1d_{1,x}-3a_{0,x}d_2-3b_1l_2p_xc_0$$
$$-6b_1c_2p_xl_1+5b_1l_2{}^2p_x{}^3l_1+15b_{2,x}p_x{}^2l_2l_1+b_{2,t}-6b_2c_1p_xl_1-3b_{2,x}c_0$$
$$-6a_2p_xl_1d_1+3b_{1,x}l_2p_{x,x}-3a_0d_1l_2p_x+b_{2,x,x,x}+b_1l_2p_{x,x,x}-6a_1p_xl_1d_2$$
$$+b_1l_2q_t-3b_{1,x}c_1+3b_{1,x,x}l_2p_x-3b_1c_{1,x}+b_1l_2p_t=0, \tag{30}$$

$$b_{1,t}+b_2p_tl_1+b_2p_{x,x,x}l_1-3a_{0,x}d_1-3b_1c_1p_xl_1-3a_1p_xl_1d_1+3b_{2,x}p_{x,x}l_1$$
$$-3b_2p_xc_0l_1-3a_0d_{1,x}+b_{1,x,x,x}+3b_{1,x}l_2p_x{}^2l_1+b_2q_tl_1+3b_{2,x,x}p_xl_1$$
$$-3b_{1,x}c_0-3b_1c_{0,x}+5b_2p_x{}^3l_2l_1{}^2+3b_1l_2p_xp_{x,x}l_1-3a_0d_2p_xl_1=0, \tag{31}$$

$$2c_2q_yl_2-2a_2p_xl_2=0, \tag{32}$$

$$c_{2,y}-a_{2,x}+c_1q_yl_2-a_1p_xl_2=0, \tag{33}$$

$$2c_2q_yl_1+c_{1,y}-2a_2p_xl_1-a_{1,x}=0, \tag{34}$$

$$-a_{0,x} + c_1 q_y l_1 - a_1 p_x l_1 + c_{0,y} = 0, \tag{35}$$

$$-2b_2 l_2 p_x + 2d_2 l_2 q_y = 0, \tag{36}$$

$$-b_{2,x} + d_{2,y} + d_1 l_2 q_y - b_1 l_2 p_x = 0, \tag{37}$$

$$d_{1,y} - b_2 p_x l_1 + d_2 q_y l_1 - b_{1,x} = 0. \tag{38}$$

From (21), (27), (32) and (36), we have

$$c_2 = {l_2}^2 {p_x}^2, \qquad a_2 = {l_2}^2 p_x q_y, \qquad d_2 = -{l_2}^{3/2} {p_x}^2, \qquad b_2 = -{l_2}^{3/2} q_y p_x. \tag{39}$$

Substituting (39) into (22), (28), (33) and (37), yields

$$b_1 = 0, \qquad a_1 = 0, \qquad c_1 = l_2 p_{xx}, \qquad d_1 = -\sqrt{l_2} p_{xx}. \tag{40}$$

Inserting (39) and (40) into (23) and (29), we obtain

$$a_0 = l_1 l_2 p_x q_y, \qquad c_0 = \frac{p_{xxx} + 2 l_1 l_2 {p_x}^3 + p_t + q_t}{3 p_x}. \tag{41}$$

Reducing (24–26), (30), (31), (34), (35) and (38) by using (39–41), we get

$$q_{ty} = 0, \tag{42}$$

from which we can derive different types of solutions. Obviously, one of special solutions is

$$q(y,t) = F(y) + c(t), \quad c \equiv c(t), \ F \equiv F(y). \tag{43}$$

From above results, the variable separation solutions of the (2+1)-dimensional ANNV system (1) possess the form

Family 1 For $l_1 l_2 = -1$,

$$\begin{aligned} u_1 = {} & -p_x F_y + p_x F_y \tanh^2_{l_1}(p + F + c) \\ & + l_1 \sqrt{l_2} p_x F_y \tanh_{l_1}(p + F + c) \operatorname{sech}_{l_1}(p + F + c), \end{aligned} \tag{44}$$

$$\begin{aligned} v_1 = {} & \frac{p_t + p_{xxx} + c_t - 2p_x^3}{3p_x} - p_{xx} \tanh_{l_1}(p + F + c) - l_1 \sqrt{l_2} p_{xx} \operatorname{sech}_{l_1}(p + F + c) \\ & + p_x^2 \tanh^2_{l_1}(p + F + c) + l_1 \sqrt{l_2} p_x^2 \tanh_{l_1}(p + F + c) \operatorname{sech}_{l_1}(p + F + c), \end{aligned} \tag{45}$$

$$\begin{aligned} u_2 = {} & -p_x F_y + p_x F_y \coth^2_{l_1}(p + F + c) \\ & + \mathrm{i} l_1 \sqrt{l_2} p_x F_y \coth_{l_1}(p + F + c) \operatorname{csch}_{l_1}(p + F + c), \end{aligned} \tag{46}$$

$$\begin{aligned} v_2 = {} & \frac{p_t + p_{xxx} + c_t - 2p_x^3}{3p_x} - p_{xx} \coth_{l_1}(p + F + c) - \mathrm{i} l_1 \sqrt{l_2} p_{xx} \operatorname{csch}_{l_1}(p + F + c) \\ & + p_x^2 \coth^2_{l_1}(p + F + c) + \mathrm{i} l_1 \sqrt{l_2} p_x^2 \coth_{l_1}(p + F + c) \operatorname{csch}_{l_1}(p + F + c). \end{aligned} \tag{47}$$

Family 2 For $l_1 l_2 = 1$,

$$u_3 = p_x F_y + p_x F_y \tan_{l_1}^2(p+F+c) - l_1\sqrt{l_2}\, p_x F_y \tan_{l_1}(p+F+c)\sec_{l_1}(p+F+c), \tag{48}$$

$$v_3 = \frac{p_t + p_{xxx} + c_t + 2p_x^3}{3p_x} + p_{xx}\tan_{l_1}(p+F+c) - l_1\sqrt{l_2}\, p_{xx}\sec_{l_1}(p+F+c) + p_x^2\tan_{l_1}^2(p+F+c) - l_1\sqrt{l_2}\, p_x^2\tan_{l_1}(p+F+c)\sec_{l_1}(p+F+c), \tag{49}$$

$$u_4 = p_x F_y + p_x F_y \cot_{l_1}^2(p+F+c) - l_1\sqrt{l_2}\, p_x F_y \cot_{l_1}(p+F+c)\csc_{l_1}(p+F+c), \tag{50}$$

$$v_4 = \frac{p_t + p_{xxx} + c_t + 2p_x^3}{3p_x} + p_{xx}\cot_{l_1}(p+F+c) - l_1\sqrt{l_2}\, p_{xx}\csc_{l_1}(p+F+c) + p_x^2\cot_{l_1}^2(p+F+c) - l_1\sqrt{l_2}\, p_x^2\cot_{l_1}(p+F+c)\csc_{l_1}(p+F+c). \tag{51}$$

Family 3 For $l_1 = 0$,

$$u_5 = \frac{2p_x F_y}{(p+F+c)^2}, \tag{52}$$

$$v_5 = \frac{p_t + p_{xxx} + c_t}{3p_x} - \frac{2p_{xx}}{p+F+c} + \frac{2p_x^2}{(p+F+c)^2}, \tag{53}$$

where p is an arbitrary functions of $\{x,t\}$, $c \equiv c(t)$ and $F \equiv F(y)$.

In additions, if we let $l_1 = -l_2 = \frac{1}{2}$ or $l_1 = l_2 = \pm\frac{1}{2}$, we will obtain the combined solutions of (1). Here, we cancel these cases for convenience.

Remark 3 It is necessary to point that all the exact solutions of the (2+1)-dimensional ANNV system constructed in this paper have been checked by Maple software. Because the mapping equation (4), we can not only get new l-deformed hyperbolic function solutions and l-deformed triangular function solutions, but also get the combined exact solutions of a class of NPDEs. To our knowledge, these solutions (43–51) have not been reported in other literatures. All the solutions (namely (44–53) of the (2+1)-dimensional ANNV system obtained in this paper include three independent variables $p(x,t)$, $c(t)$ and $F(y)$. In these solutions the arbitrary functions imply that (1) has abundant local physical structures. Since these similar local physical structures have been widely discussed in some previous literatures [1–14], the related plots are neglected in our present paper.

4 Summary and Discussion

In summary, the extended tanh-function method is improved to obtain variable separation solutions of (2+1)-dimensional ANNV system, and some lower dimensional arbitrary functions can be included in their exact solutions. Actually, our present short paper is merely a beginning work, and we can obtain richer exact solutions by a more general ansatz of

NPDE (2), which reads

$$u = a_0(x) + \sum_{j=1}^{n}\left\{a_j(x)\phi[R(x)]^j + \frac{b_j(x)}{\phi[R(x)]^j} + c_j(x)\phi[R(x)]^{j-1}\sqrt{l_1 + l_2\phi[R(x)]^2}\right.$$
$$\left. + \frac{d_j(x)}{\phi[R(x)]^{j-1}\sqrt{l_1 + l_2\phi[R(x)]^2}}\right\}. \tag{54}$$

References

1. Tang, X.Y., Lou, S.Y., Zhang, Y.: Phys. Rev. E **66**, 046601 (2002) (and references therein)
2. Zheng, C.L., Fang, J.P., Chen, L.Q.: Znaturforsch **59a**, 912 (2004)
3. Dai, C.Q., Zhou, G.Q., Zhang, J.F.: Chaos Solitons Fractals **33**, 1458 (2007)
4. Dai, C.Q., Liu, F.Q., Zhang, J.F.: Novel types of interactions between solitons in the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov system. Chaos Solitons Fractals (2006). doi:10.1016/j.chaos.2006.06.070
5. Zhu, H.P., Fang, J.P., Zheng, C.L.: Commun. Theor. Phys. **45**, 127 (2006)
6. Dai, C.Q., Yan, C.J., Zhang, J.F.: Commun. Theor. Phys. **46**, 389 (2006)
7. Dai, C.Q., Zhang, J.F.: J. Math. Phys. **47**, 043501 (2006)
8. Fang, J.P., Zheng, C.L.: Chin. Phys. **14**, 669 (2005)
9. Fang, J.P., Ren, Q.B., Zheng, C.L.: Znaturforsch **60a**, 245 (2005)
10. Zhu, H.P., Zheng, C.L., Fang, J.P.: Phys. Lett. A **355**, 39 (2006)
11. Dai, C.Q., Zhou, G.Q.: Chin. Phys. **16**, 1201 (2007)
12. Dai, C.Q., Zong, F.D., Zhang, J.F.: Int. J. Theor. Phys. **45**, 813 (2006)
13. Dai, C.Q., Chen, R.P.: Znaturforsch **61a**, 307 (2006)
14. Dai, C.Q., Ni, Y.Z.: Phys. Scr. **74**, 584 (2006)
15. Dai, C.Q.: Phys. Scr. **75**, 310 (2007)
16. Boiti, M., Leon, J.J.P., Manna, M., Pempinelli, F.: Inverse Probl. **2**, 271 (1986)
17. Lou, S.Y., Hu, X.B.: J. Math. Phys. **38**, 6401 (1997)
18. Estévez, P.G., Leble, S.: Inverse Probl. **11**, 925 (1996)
19. Ablowitz, M.J., Clarkson, P.A.: In: Solitons, Nonlinear Evolution Equations and Inverse Scattering. Lecture Notes Series, vol. 149. Cambridge University Press, Cambridge (1991)
20. Leble, S.B., Ustinov, N.V.: Inverse Probl. **10**, 617 (1991)
21. Hirota, R., Satsuma, J.: J. Phys. Soc. Jpn. **40**, 611 (1994)
22. Hu, H.C., Tang, X.Y., Lou, S.Y., Liu, Q.P.: Chaos Solitons Fractals **22**, 327 (2004)
23. Arai, A.: J. Math. Anal. Appl. **158**, 63 (1991)
24. Li, B., Chen, Y., Zhang, H.Q.: Czech. J. Phys. **53**, 283 (2003)